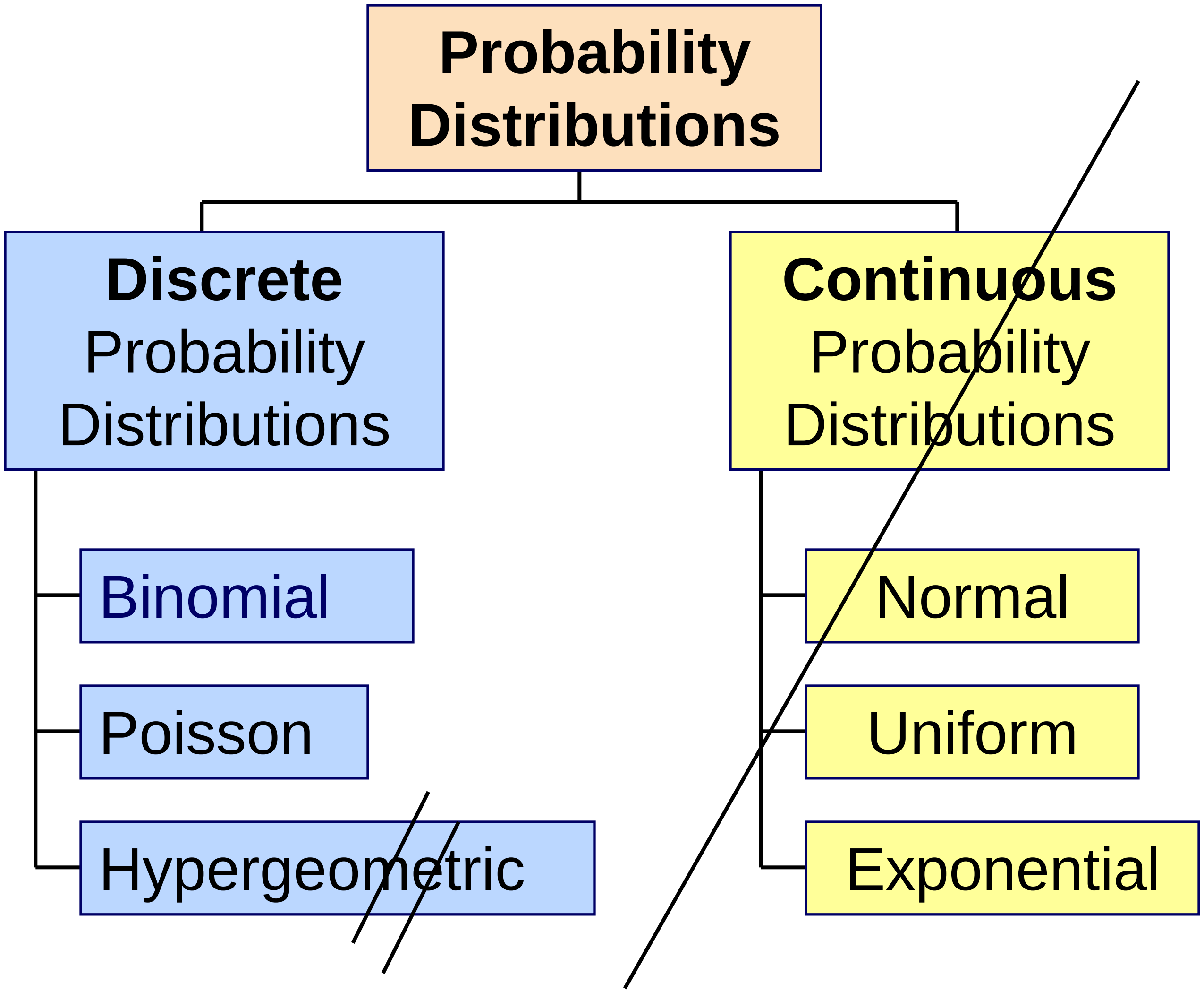
Discrete
Probability Distributions

Probability Distributions



Discrete Probability Distributions

- A discrete random variable is a variable that can assume only a countable number of values

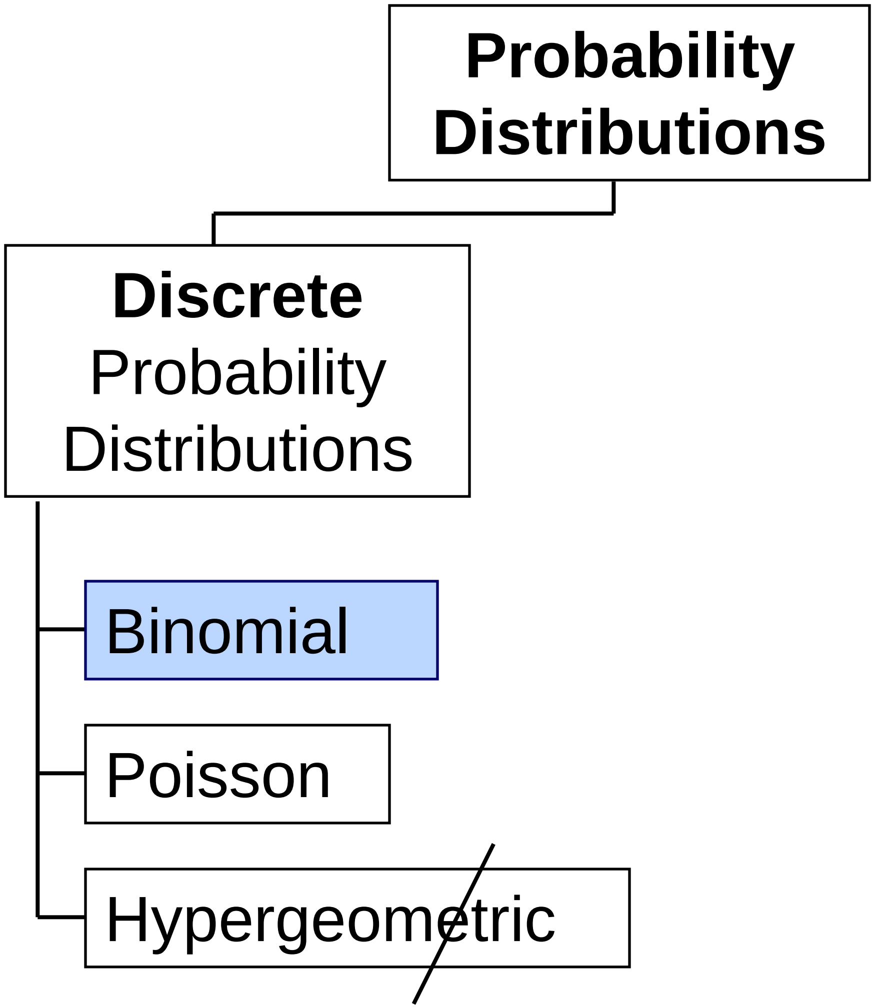
Many possible outcomes:

- number of complaints per day
- number of TV's in a household
- number of rings before the phone is answered

Only two possible outcomes:

- gender: male or female
- defective: yes or no
- spreads peanut butter first vs. spreads jelly first

The Binomial Distribution



The Binomial Distribution

- Characteristics of the Binomial Distribution:
  - A trial has only two possible outcomes – “success” or “failure”
  - There is a fixed number, n, of identical trials
  - The trials of the experiment are independent of each other
  - The probability of a success, p, remains constant from trial to trial
  - If p represents the probability of a success, then $(1-p) = q$ is the probability of a failure

Binomial Distribution Settings

- A manufacturing plant labels items as either defective or acceptable
- A firm bidding for a contract will either get the contract or not
- A marketing research firm receives survey responses of “yes I will buy” or “no I will not”
- New job applicants either accept the offer or reject it

Counting Rule for Combinations

- A combination is an outcome of an experiment where x objects are selected from a group of n objects

$$C_x^n = \frac{n!}{x!(n-x)!}$$

where:

$n! = n(n - 1)(n - 2) \ldots (2)(1)$

$x! = x(x - 1)(x - 2) \ldots (2)(1)$

$0! = 1$ (by definition)

Binomial Distribution Formula

$$P(x) = \frac{n!}{x!(n-x)!} p^{x} q^{n-x}$$

P(x) = probability of **x** successes in **n** trials,
with probability of success **p** on each trial

x = number of 'successes' in sample, (x = 0, 1, 2, ..., n)

p = probability of "success" per trial

q = probability of "failure" = (1 – p)

n = number of trials (sample size)

Example: Flip a coin four times, let x = # heads:

n = 4

p = 0.5

q = (1 - .5) = .5

x = 0, 1, 2, 3, 4

Binomial Distribution Characteristics

- Mean

$$\mu = E(x) = np$$

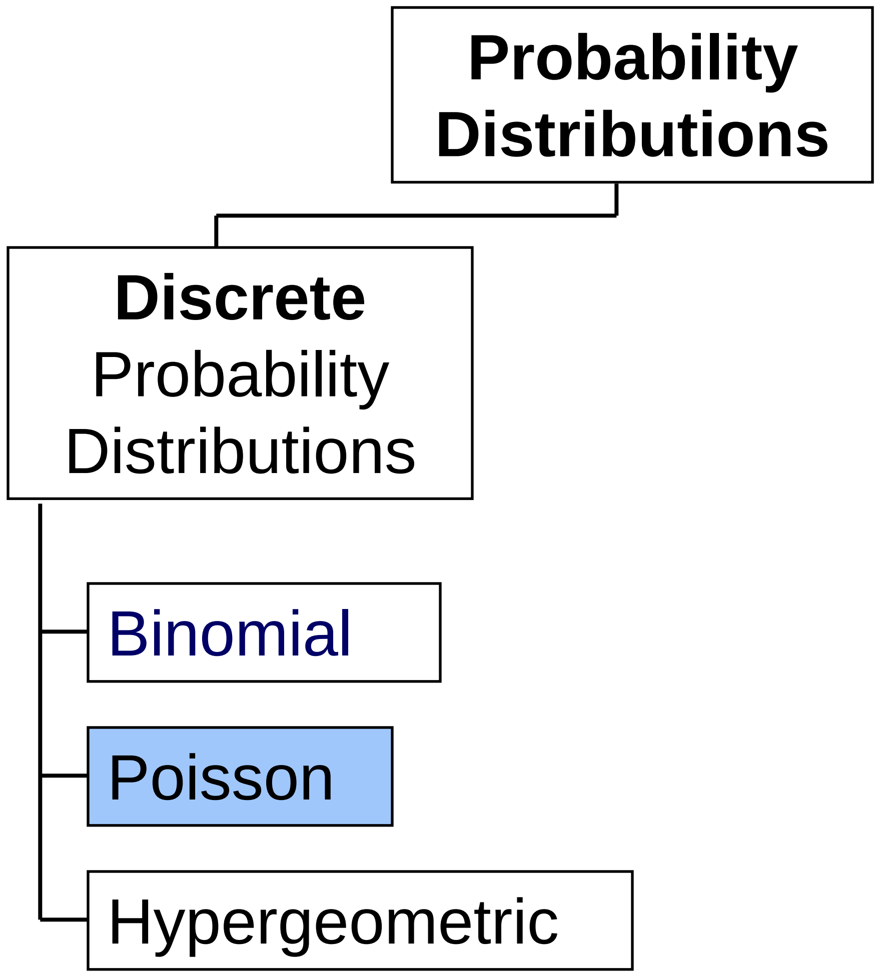
- Variance and Standard Deviation

$$\sigma^2 = npq$$

$$\sigma = \sqrt{npq}$$

Where n = sample size
p = probability of success
q = (1 – p) = probability of failure

The Poisson Distribution



The Poisson Distribution

- Characteristics of the Poisson Distribution:
  - The outcomes of interest are rare relative to the possible outcomes
  - The average number of outcomes of interest per time or space interval is λ
  - The number of outcomes of interest are random, and the occurrence of one outcome does not influence the chances of another outcome of interest
  - The probability of that an outcome of interest occurs in a given segment is the same for all segments

Poisson Distribution Formula

$$P(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

where:

t = size of the segment of interest

x = number of successes in segment of interest

λ = expected number of successes in a segment of unit size

e = base of the natural logarithm system (2.71828...)

Poisson Distribution Characteristics

- Mean

$$\mu = \lambda t$$

- Variance and Standard Deviation

$$\sigma^2 = \lambda t$$

$$\sigma = \sqrt{\lambda t}$$

where λ = number of successes in a segment of unit size

t = the size of the segment of interest

Please practice sample problems

End